

Roadster Thrusters

I have attached the charts separately.

These notes are my working notes.

Megan E S Macafee

Set Up and Assumptions

Let us assume that the car's traction management system uses static friction (i.e. no slipping or water) and does not fall-back to kinetic friction - typically 25% less acceleration. So $\mu = 1.0$ (Cast Iron/copper $\mu \sim 1.05$ / ice/ice ~ 0.02).

https://en.wikipedia.org/wiki/Gravitational_acceleration

```
In[*]:= g = 9.80665;  
oneInch = 2.54 × 10-2; (* m *)  
oneMilePerHour =  $\frac{5280 \times 12 \text{ oneInch}}{60^2}$  (*  $\frac{\text{m}}{\text{s}}$  *);  
sixtyMPH = 60 oneMilePerHour
```

```
Out[*]:= 26.8224
```

```
In[*]:= oneHundredMPH = 100 oneMilePerHour
```

```
Out[*]:= 44.704
```

```
In[*]:= AvailableAcceleration[carMass_] := Module[{μstatic = 1.0, downForce, forwardForce},  
  (* no aerodynamics for the moment *)  
  downForce = g carMass;  
  forwardForce = μstatic downForce;  
  (* in the simple case with no aerodynamics the mass CANCELS OUT *)  
   $\frac{\text{forwardForce}}{\text{carMass}}$   
]
```

It is actually a lot more complicated than this <http://theracingline.net/2018/race-car-tech/race-tech-explained/-tyres-combined-tyre-forces/>

```
In[*]:= estRoadsterMass = 2300 (* guessed from 2300 kg Model X *)
```

```
Out[*]:= 2300
```

```
In[*]:= carLength = 179 oneInch;  
carWidth = 82 oneInch;  
carArea = carLength carWidth (* Again all of these are guessed using the Model X data*)
```

```
Out[*]:= 9.46966
```

```
In[*]:= standardAtmosphere = 101325 (* Newtons per m2 *)
```

```
Out[*]:= 101325
```

Without Thrusters Normal

$$v = u + at$$

$$u = 0$$

$$v = at \text{ (in our case) and } t = \frac{v}{a}$$

```
In[*]:= time60 =  $\frac{\text{sixtyMPH}}{\text{AvailableAcceleration}[\text{estRoadsterMass}]}$  (* 1.9 seconds quoted *)
```

```
Out[*]:= 2.73512
```

```
In[*]:= time100 =  $\frac{\text{oneHundredMPH}}{\text{AvailableAcceleration}[\text{estRoadsterMass}]}$  (* 4.2 seconds quoted *)
```

```
Out[*]:= 4.55854
```

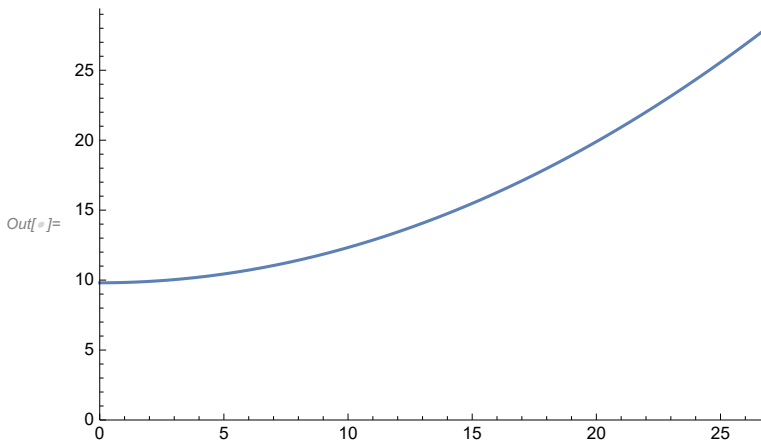
```
In[*]:= AvailableAcceleration[carMass_, velocity_] :=
  Module[{μstatic = 1.0, downForce, forwardForce, unknown = 58},
    (* no aerodynamics for the moment *)
    downForce = g carMass + unknown velocity2;
    (* I looked at other aerodynamic structures and made this guess *)
    forwardForce = μstatic downForce;

     $\frac{\text{forwardForce}}{\text{carMass}}$ 
  ]
```

```
In[*]:= Expand[AvailableAcceleration[estRoadsterMass, vel]]
```

```
Out[*]:= 9.80665 + 0.0252174 vel2
```

```
In[*]:= Plot[AvailableAcceleration[estRoadsterMass, v],
  {v, 0, sixtyMPH}, PlotRange → {{0, sixtyMPH}, {0, All}}]
```



```

In[*]:= time60claim = 1.9;
time100claim = 4.2;

position = 0; velocity = 0; step = 0.01;
history = Append[{}, {0, position, velocity}];

Do[
  position = position + velocity step;
  velocity = velocity + AvailableAcceleration[estRoadsterMass, velocity] step;
  AppendTo[history, {t + step, position, velocity}],
  {t, 0, time60claim, step}
];

velocityPlot = ListLinePlot[
  Map[#[[{1, 3}]] &, history],
  PlotLabel → "Velocity and Distance vs. Time",
  PlotStyle → Directive[Thick],
  PlotRange → {{-.15, time60claim 1.1}, {0, sixtyMPH 1.1}},
  AxesLabel → {"Time\n(Seconds)", "Velocity and Distance\n(m/s and m)"},
  ImageSize → 700
];

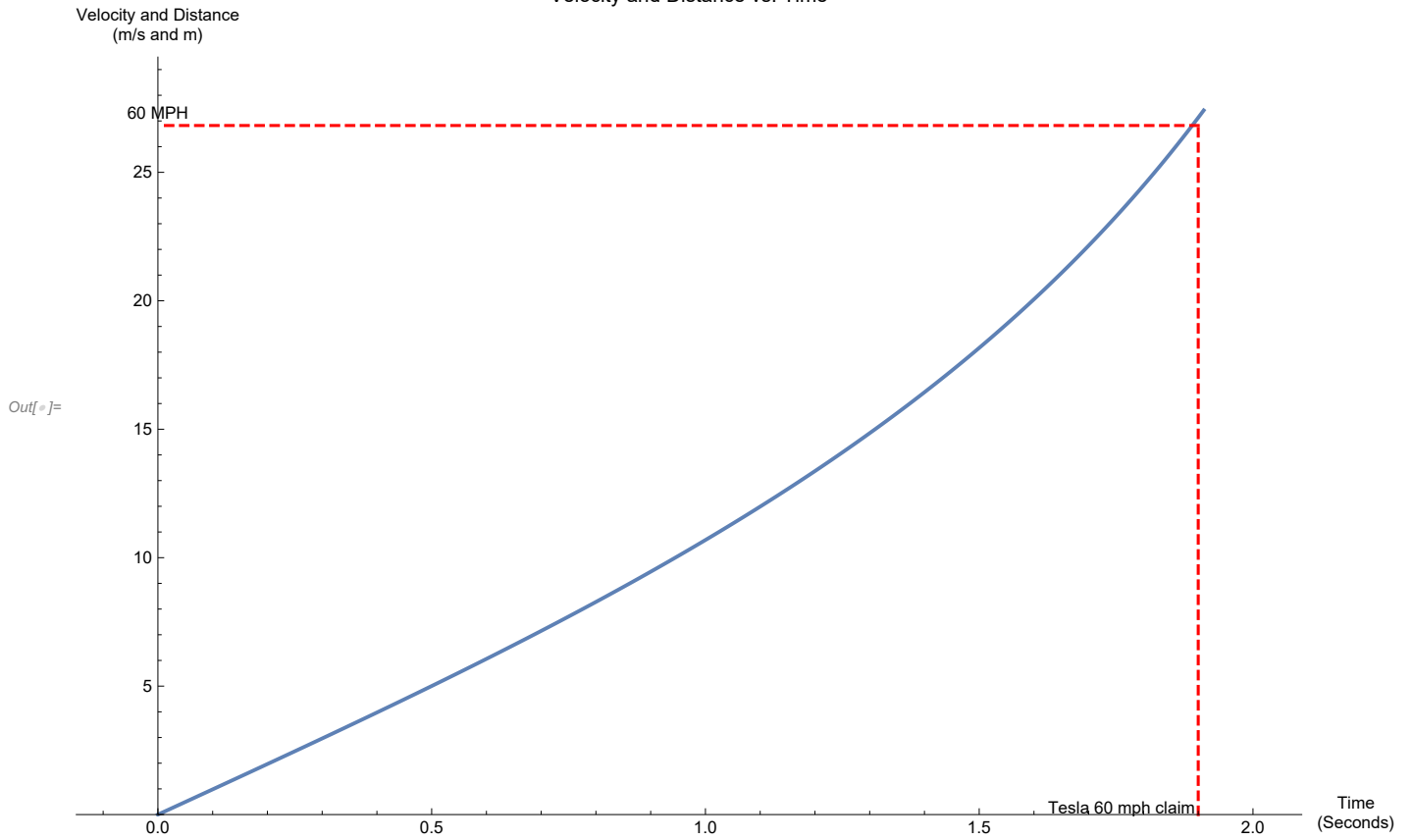
sixtyPlot = ListLinePlot[
  {
    Labeled[{time60claim, 0}, "Tesla 60 mph claim"],
    {time60claim, sixtyMPH}
  },
  PlotStyle → Directive[Red, Dashed]
];

speedPlot60 = ListLinePlot[
  {{time60claim, sixtyMPH}, Labeled[{0, sixtyMPH}, "60 MPH", {Top}]},
  PlotStyle → Directive[Red, Dashed]];

Show[{
  velocityPlot, sixtyPlot, speedPlot60
}]

```

Velocity and Distance vs. Time



In[*]:= peakAeroForceGuess = 58 sixtyMPH²

Out[*]:= 41 727.6

In[*]:= pressureDifference = $\frac{\text{peakAeroForceGuess}}{\text{carArea}}$ (* Newtons/m² *)

Out[*]:= 4406.45

In[*]:= percentOfAtmos = $\frac{\text{pressureDifference}}{\text{standardAtmosphere}}$ 100 (* % of one atmosphere *)

Out[*]:= 4.34883

So, we have model that gives the right 0-60 time if the 60 mile per hour downforce is about 4.5 tonnes - which suggest that the suction at 60 mph is 4% of atmospheric.

The 0-100 mph times are depressed by comparison (they do not continue to grow as fast as the curve above) - I assume that the engines are not powerful enough to stay on the upper limit of the sticky-friction curve?

Comparison Data

```
In[*]:= MPHPosition[lineList_] := Position[lineList, "mph"][[1, 1]];
```

```
zeroSixtyWebData = Import["https://www.0-60specs.com/0-60-times/"];
```

```
... $CharacterEncoding: The byte sequence {240} could not be interpreted as a character in the UTF-8 character encoding.
```

```
... $CharacterEncoding: The byte sequence {159} could not be interpreted as a character in the UTF-8 character encoding.
```

```
... $CharacterEncoding: The byte sequence {153} could not be interpreted as a character in the UTF-8 character encoding.
```

```
... General: Further output of $CharacterEncoding::utf8 will be suppressed during this calculation.
```

```
In[*]:= goodLines = Select[StringSplit[zeroSixtyWebData, "\n"], StringContainsQ[#, "@"] &];
```

```
In[*]:= wordListInLineList = Map[
  StringSplit,
  goodLines
];
```

```
plotDataRaw = Map[{
  StringRiffle[#[[1 ;; MPHPosition[#] - 10]]],
  ToExpression[#[[MPHPosition[#] - 9]]],
  ToExpression[#[[MPHPosition[#] - 6]]]
} &,
wordListInLineList
];
```

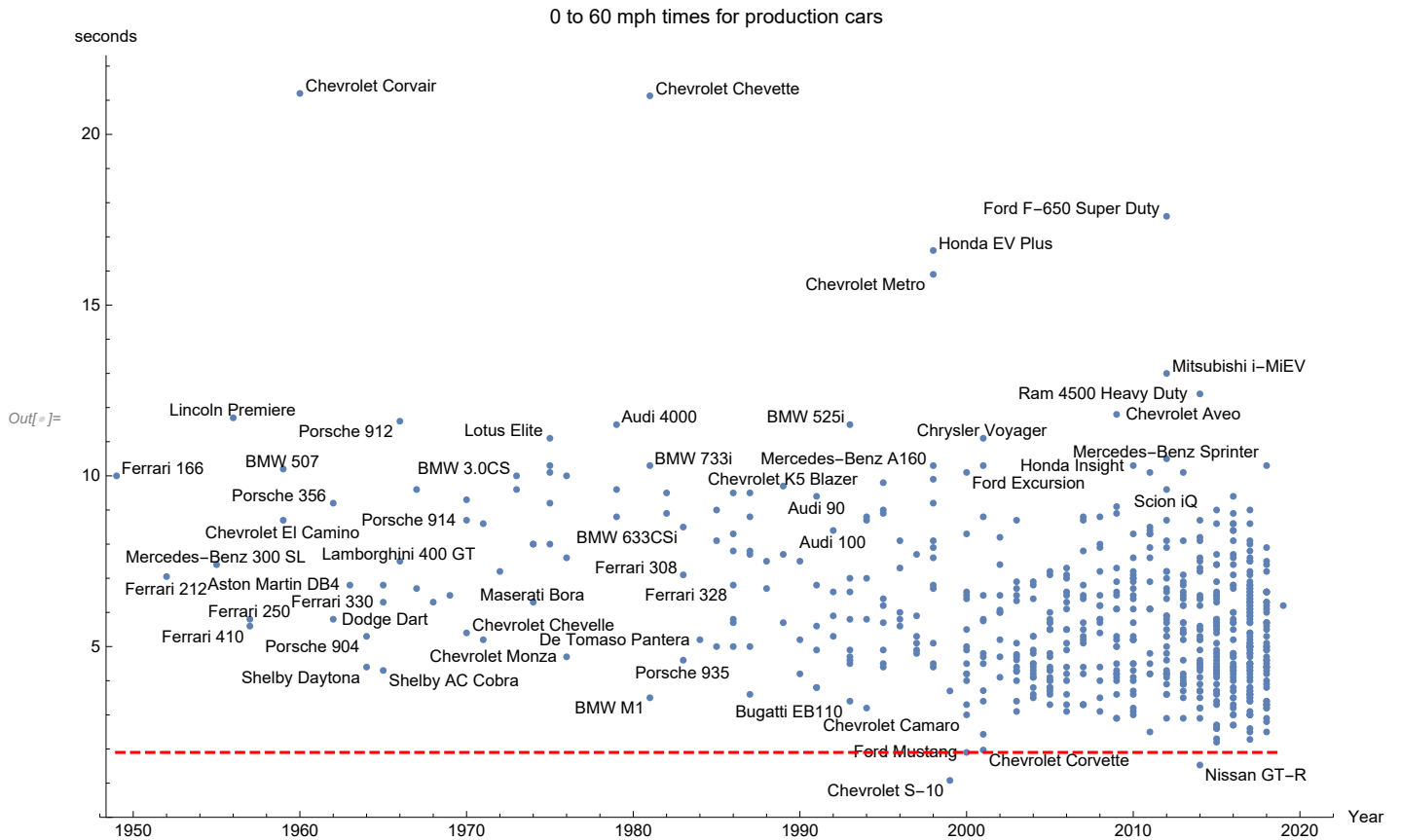
```
plotData = Select[plotDataRaw, Between[#[[2]], {1949, 2019}] && Between[#[[3]], {0, 500}] &];
```

ToExpression::sntxi: Incomplete expression; more input is needed .

```
In[*]:= {firstYear, lastYear} = MinMax[Map[#[[2]] &, plotData]]
```

```
Out[*]:= {1949, 2019}
```

```
In[ ]:= Show[ {
  ListPlot[
    Map[Labeled[#[[2 ;;]], #[[1]]] &, plotData],
    PlotLabel -> "0 to 60 mph times for production cars",
    AxesLabel -> {"Year", "seconds"},
    PlotRange -> {{firstYear - 1, lastYear + 3}, {0, All}},
    ImageSize -> 700
  ],
  ListLinePlot[{{firstYear, time60claim}, {lastYear, time60claim}},
    PlotStyle -> Directive[{Red, Dashed}]
  ]
}
```



The very fast cars are getting help with their acceleration probably using aerodynamics.

With Thrusters (In addition to existing aerodynamic forces and friction)

$$v = u + at$$

$$u = 0$$

$$v = at \text{ (in our case) and } t = \frac{v}{a}$$

```

In[*]:= ThrusterAcceleration[carMass_, velocity_, fractionAtmosphere_] :=
Module[{μstatic = 1.0, downForce, forwardForce, unknown = 58, suction},
  suction = standardAtmosphere carArea fractionAtmosphere (* fractionAtmosphere atmospheres *);
  (* no aerodynamics for the moment *)
  downForce = g carMass + unknown velocity2 + suction;
  forwardForce = μstatic downForce;

  forwardForce
  carMass
]

```

```

In[*]:= Expand[ThrusterAcceleration[estRoadsterMass, vel, 0.1]]

```

```

Out[*]:= 51.5246 + 0.0252174 vel2

```

```

In[*]:= Expand[ThrusterAcceleration[estRoadsterMass, vel, 0]]

```

```

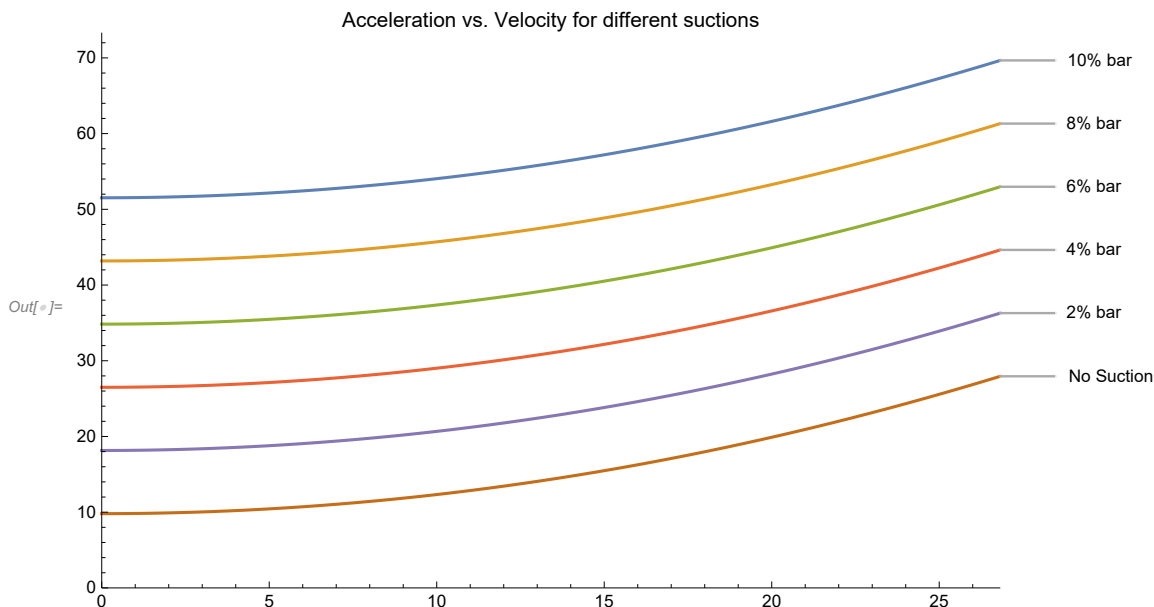
Out[*]:= 9.80665 + 0.0252174 vel2

```

```

In[*]:= Plot[{
  ThrusterAcceleration[estRoadsterMass, v, .1],
  ThrusterAcceleration[estRoadsterMass, v, .08],
  ThrusterAcceleration[estRoadsterMass, v, .06],
  ThrusterAcceleration[estRoadsterMass, v, .04],
  ThrusterAcceleration[estRoadsterMass, v, .02],
  ThrusterAcceleration[estRoadsterMass, v, 0]
}, {v, 0, sixtyMPH},
PlotRange → {{0, sixtyMPH}, {0, All}},
PlotLabel → "Acceleration vs. Velocity for different suctions",
ImageSize → Large,
PlotLabels → {"10% bar", "8% bar", "6% bar", "4% bar", "2% bar", "No Suction"}
]

```



```

In[ ]:= position = 0; velocity = 0; step = 0.01;
history = Append[{}, {0, position, velocity}];

suctionGenerated = .03; (* fraction of an atmosphere *)
suctionDuration = .5; (* duration of suction *)

Do[
  position = position + velocity step;
  velocity =
  velocity +
  If[
    t ≤ suctionDuration,
    ThrusterAcceleration[estRoadsterMass, velocity, suctionGenerated],
    ThrusterAcceleration[estRoadsterMass, velocity, 0]
  ] ×
  step;
  AppendTo[history, {t + step, position, velocity}],
  {t, 0, 1.29, step}
];

suctionVelocityPlot = ListLinePlot[
  Map[#[[{1, 3}]] &, history],
  PlotLabel → "Velocity and Distance vs. Time",
  PlotStyle → Directive[Thick],
  PlotRange → {{-.15, time60claim 1.1}, {0, sixtyMPH 1.1}},
  AxesLabel → {"Time\n(Seconds)", "Velocity and Distance\n(m/s and m)"},
  PlotTheme → "Detailed",
  ImageSize → Large
];

sixtyPlot = ListLinePlot[
  {
    Labeled[{time60claim, 0}, "Tesla 60 mph claim"],
    {time60claim, sixtyMPH}
  },
  PlotStyle → Directive[Red, Dashed]
];

speedPlot60 = ListLinePlot[
  {{time60claim, sixtyMPH}, Labeled[{0, sixtyMPH}, "60 MPH", {Top}]},
  PlotStyle → Directive[Red, Dashed]];

Show[{
  suctionVelocityPlot, velocityPlot, sixtyPlot, speedPlot60
}]

```